Harmonic Beam Steering in Time–Modulated Arrays with Simultaneous Sidelobe Control

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Abstract—In this study, a methodological beam steering approach in time-modulated linear arrays with simultaneous beam sidelobe control is presented. The method is based on the difference between pulse amplitude in order to provide necessary phase shift to steer the main beam and control the sidelobe level in harmonic frequencies. In order to illustrate the asserted idea a 16 element linear array placed along z-axis with zero phase is considered. The results show that the proposed approach is an effective method in both beam steering and sidelobe level control.

Index Terms—4D arrays, time-modulation, linear arrays, beam steering.

I. INTRODUCTION

BESIDE the switched array concept which is originally proposed in late 50’s by Shank and Bickmore [1] brings an additional degree of freedom in low/ultralow sidelobe array design, due to periodic switching of array elements, a seperation of radiated power between the main operation frequency and the harmonic frequencies called sideband radiations becomes inevitable. At first, these radiations are tried to be reduced as much as possible in order to shift the radiated power into main operation frequency via evolutionary techniques [2]–[4] and theoretical researches have conducted on the calculation and the control of these radiations [5]–[11].

In the mentioned above works, the sidebands are tried to be suppressed since they are regarded as power loss, however, Tennant and Chambers asserted that these radiations may be used in direction finding (DF) applications [12] and this idea is verified in [13]. In this way, the idea of exploiting the sidebands comes forward. Since the sideband usage in DF applications has been shown in [12], the pattern shaping in these frequencies for the usage of these radiations becomes inevitable. Hence, in order to communicate over the fundamental radiation, an evolutionary approach has been introduced by Li et. al. to shape and steer the fundamental harmonic beam [14]. Also, a scale and shift method has been applied to the same problem of harmonic beam steering by Tong and Tennant [15].

In this study, a harmonic beam steering methodological technique with sidelobe control based on pulse difference is introduced and in order to show the effectiveness of the technique an 16 element linear array is given as an explanatory example.

II. TIME MODULATION AND PULSE DIFFERENCE

Suppose that each element of an antenna array is switched periodically by some simple on-off switches. For the variable aperture size (VAS), this switching action switching consisting of ideal pulses in one switching period may be modelled as:

\[ U_n(t) = \begin{cases} 1, & 0 < t \leq t_n \leq T_p \\ 0, & \text{otherwise} \end{cases} \]  

where \( t_n \) represents the finishing instant of the pulse and \( T_p \) represents the modulation period. Since, the switching process in periodic in time the pulse train \( U_n \) can be decomposed into complex Fouries series (CFS) and for the VAS time scheme CFS is given by:

\[ U_n(t) = \sum_{m=-\infty}^{\infty} C_{mn} e^{j mw_p t} \]  

where \( w_p \) is the angular switching frequency (i.e. \( w_p = \frac{2\pi}{T_p} \)) and \( C_{mn} \) is the complex Fourier coefficients (CFC) defined as:

\[ C_{mn}(t) = \frac{1}{T_p} \int_{0}^{T_p} U_n(t) e^{-j mw_p t} dt. \]  

Additionally, for VAS time scheme, the CFCs may be written as:

\[ C_{mn} = \frac{1}{jm2\pi} [1 - e^{-j m 2\pi \tau_n}], \]

where \( m \) represents the harmonic number and \( \tau_n \) represents the normalized switch-on duration of element number \( n \) (i.e. \( \tau_n = t_n/T_p \)).

It can easily be shown that at a far field observation point, the Poynting vector of a time modulated array tends to unmodulated array’s Poynting vector multiplied by a CFC factor, if the carrier frequency is much bigger than the switching frequency (i.e. \( f_0/f_p \gg 1 \)). Hence, the total time averaged array factor of an \( N \) element time modulated linear array consisting of isotropic sources and positioned along positive \( z \)-axis may be written as:

\[ F(\theta) = \sum_{m=-\infty}^{\infty} \sum_{n=1}^{N} I_n C_{mn} e^{j k d_n \cos(\theta)} \]

where \( I_n, d_n \) and \( k \) represent the complex excitation amplitude, the relative distance between \( n \)-th element and phase center of the array and wavenumber (i.e. \( k = \frac{2\pi}{\lambda_0} \)) at operating frequency, respectively. Additionally, \( m \) represents the harmonic number and \( m = 0 \) term represents array factor.
the central operating band and \(|m| \geq 1\) terms represent the sideband radiations. From (5) it is clear that at central operating frequency the array factor contains an additional multiplier which may be used in design. Moreover, it must be noted that (5) is valid only under far field and \(f_0/f_p \gg 1\) approximations.

Now, suppose that each switch end is connected to an amplifier (or attenuator) that controls the relative pulse level. For this type switching, one pulse in one period may be modelled as:

\[
U_n(t) = \begin{cases} 
K_n^1, & 0 < t \leq t_n \leq T_p \\
K_n^2, & t_n \leq t \leq T_p 
\end{cases},
\]

which represent two adjacent pulses that cover whole modulation period. Since the pulse scheme is the linear combination of two pulses, each pulse may be decomposed into CFS independently and may be combined afterwards. Hence, the CFCs for this type switching may be written as:

\[
C_{mn} = \frac{1}{jm2\pi} \left[ \left( (K_n^1 - K_n^2) - (K_n^1 - K_n^2) e^{-jm2\pi \tau_n} \right) \right],
\]

where \((K_n^1 - K_n^2)\) represents the amplitude difference between the adjacent pulses and may be represented by one variable \(\Delta\) (i.e. \(\Delta = (K_n^1 - K_n^2)\)). Hence, the CFC’s may be written as:

\[
C_{mn} = \frac{\Delta}{jm2\pi} \left[ 1 - e^{-jm2\pi \tau_n} \right].
\]

For the fundamental harmonic frequency this coefficient becomes:

\[
C_{1n} = \frac{\Delta}{\pi} \sin(\pi \tau_n) e^{-j\pi \tau_n}
\]

and it is clear that the normalized switch on duration appears as a phase term for harmonic frequencies which may be used to steer the main beam of the related pattern and \(\Delta\) appears as a amplitude control variable which may be used to control the sidelobes.

### III. Beam Steering and Sidelobe Control

If the time scheme given in (1) is considered for any arbitrary \(\tau_n\) the phase varies between \(\beta \in [0, \pi]\) and this situation prevents a complete beam steering in elevation plane. However, for the proposed switching function given in (6), the sign condition of \(\Delta\) provides an additional \(\pi\) phase shift which transforms the range of phase from \([0, \pi]\) to \([\pi, 2\pi]\) and completes the blank in the phase range of \([0, 2\pi]\). Hence, the complete elevation scan becomes achievable with this type scheming by controlling the switch-on durations.

Besides the normalized switch-on duration \(\tau_n\) appears as a phase term in array factor, it is also the argument of the sine function in the amplitude terms. Since the \(\tau_n\) values are determined for the beam steering, it also defines the amplitude terms and for sidelobe reduction an additional control is necessary. Here, the pulse difference comes forward and provides the necessary parameter to control the sidelobe level.

Here one additional problem is the zero phase-null amplitude condition. Once the phase terms are calculated there may occur zero phases which makes the corresponding switch-on duration becomes zero means that the related element is forced to be turned-off. To avoid this condition once the phase terms are calculated, an additional phase offset \(P\) may be added to phase terms. This offset value may be any real value but since the phases are periodic with \(2\pi\) and the \(\tau_n\) varies in the range \(\tau_n \in [0, 1]\), the offset value may be taken in the range \(P \in [0, 1]\). Finally, the proposed method for the fundamental frequency may be summarized as follows:

- Calculate the progressive phase terms \((\beta)\) as in conventional unmodulated arrays.
- Offset these values by \(P \in (0, 1]\) in modulo \(2\pi\) (i.e. \(\beta' = \mod_{2\pi}(\beta + P)\)).
- Check whether the phase terms are below \(\pi\) or above \(\pi\).
- Set \(\tau_n^1\) as \((\tau_n^1 = \beta' / \pi)\) and \(\tau_n^2 = 1 - \tau_n^1\) for the phase terms below \(\pi\) \((K_n^1 > K_n^2)\).
- Set \(\tau_n^1\) as \((\tau_n^1 = \beta' - 1)/\pi\) and \(\tau_n^2 = 1 - \tau_n^1\) for the phase terms above \(\pi\) \((K_n^1 < K_n^2)\).
- Choose an appropriate weighting \(W_n\) to reduce sidelobe (e.g. an appropriate Taylor distribution or an optimized weightening).
- Calculate the pulse difference \(\Delta_n = \pi W_n / \sin(\pi \tau_n)\) and normalize \(\Delta\) values with \(\max(\Delta_n)\).
- Set the bigger \(K_n\) to unity between \(K_n^1\) and \(K_n^2\) (i.e. \(K_n^1 = 1\) if \(K_n^1 > K_n^2\) or vice versa).
- Set lower \(K_n\) as \(K_n = 1 - \Delta_n\).

To illustrate the proposed method, consider a 16 element linear array placed along z-axis consisting of isotropic sources with zero phases and half wavelength interelement spacing. The first harmonic is wanted to be steered to 40° in elevation with -30 dB sidelobe level. To achieve this condition a -30dB Chebyshev distribution steered to 40° is given in Fig. 1. In Fig. 1 the darker color in the pulse duration represents a higher amplitude level with respect to lighter one. The amplitude levels of the pulses given in Fig. 1 are given in Fig. 2 and the combination of the both figures represent the resultant switching scheme for -30 dB Chebyshev distribution steered to 40° in elevation. The normalized radiation pattern at the fundamental harmonic frequency is also given in Fig. 3. As it can be seen from Fig. 3 that the proposed method successfully handle both the steering and sidelobe control operations. Hence, it can be said that both steering and sidelobe control may be achieved without using time consuming evolutionary optimizers or scaling the pulse durations to small values which reduces both the directivity and the time average radiated power at that frequency.

### IV. Conclusion

In this study, a methodological beam steering approach at fundamental harmonic frequency in time modulated linear arrays is presented. A procedure based on the the combination of the difference between relative amplitude levels of
adjacent pulses and well known conventional array techniques is defined to steer the main beam and reduce the sidelobe in time-modulated linear arrays. A 16 element linear array example is given to demonstrate the proposed method and from the results obtained the proposed alternative technique successfully handle both the beam steering and sidelobe level control operations at fundamental harmonic.

REFERENCES